Open Access Research Journal of
Science and Technology
Journals home page: https://oarjst.com/
ISSN: 2782-9960 (Online)

OARJ OPEN ACCESS RESEARCH JOURNALS

(RESEARCH ARTICLE)

Check for updates

A Topological-Rough Algebraic Framework: Extending Approximations to Ring and Fields

Nassir Ali Zubain *

Department of Mathematical, Open Educational College. Wasit, Iraq.

Open Access Research Journal of Science and Technology, 2025, 15(02), 119-128

Publication history: Received on 25 October 2025; revised on 04 December 2025; accepted on 07 December 2025

Article DOI: https://doi.org/10.53022/oarjst.2025.15.2.0141

Abstract

The important effort of Pawlak established Rough Set Theory to robust paradigm for handling uncertainty by defining approximation of concepts within an equivalence-based approximation space (U, R). Subsequent research has explored the connections between rough sets, algebraic structures, and topology. Though, a unified framework that seamlessly integrates these three domains for complex algebraic structures like rings and fields remains largely undeveloped.

This paper introduce a novel hybrid structure, the Topological-Rough Algebraic Framework, which generalizes the concept of a topological approximation space to the realm of rings and fields. We construct an approximation space $(R, \equiv_I)$ where the equivalence relation $\equiv_I$ is induce for Ideal I of a Ring R, partitioning to Ring into cossets. This partition naturally serves as a aim for a Topology τ_I to R, leading to the integrated triple $(R, \equiv_I, \tau_I)$,

Within the proposed framework we investigate lower and upper approximation of subsets, characterizing the conditions under which a set is exact or rough. We further analyze key topological properties of the space (R, τ_I) establishing their relationships with the underlying algebraic properties of the ring and its ideals. The framework not only provides a theoretical generalization of existing models but also demonstrates practical applicability in coding theory, cryptographic analysis, and data mining. This work bridges a significant gap between algebraic structures, rough set theory, and general topology, offering a powerful new lens for mathematical analysis.

Keywords: Rough Sets; Topological Approximation Space; Rings; Ideals; Coset topology; Algebraic-Structures; Data Analysis

1. Introduction

The problem of uncertainty and granularity in data has been a fundamental challenge across various scientific disciplines. To address this, Pawlak (1992) introduced Rough. Set. The$o$ry as a mathematically sound frame on reasoning about imprecise knowledge [1]. The main of this concept fabrications in the approximation space (U, R), where an equivalence relation R on a universe U is used to define lower and upper approximations of any subset $X \subseteq U$, effectively separating its certain from its possible elements [1,2]. This seminal work opened doors for handing vagueness in info methods.

The evolution of rough set theory has been marked by fruitful intersections with other mathematical fields. A significant trajectory has been its fusion with algebraic structures. Biswas and Nanda (1994) pioneered this direction by introducing the notions of Ro$u$gh (Gr$o$ups and Subgr$o$ups) [3]. This line of inquiry was substantially advanced by Davvaz (2019), who systematically explored roughness within the context of rings, utilizing ideals to define approximation relations [4]. Recent work by Miao et al. (2021) further demonstrates the modern relevance of using ideals in lattices for rough set approximations [5].

* Corresponding author: Nassir Ali Zubain

At the same time as, a parallel stream of research has with topology. Wiweger (1989) was among the first to explicitly demonstrate that the families of definable sets in a Pawlak approximation space constitute a topology [6]. This established the foundational concept of a topological approximation space (u, R, τ). Where topological operators like interior and closure align by the Lo$w$er and Upper approx$i$mations, respectively [6,7]. The standard reference for the topological supporting, such as Munkres (2000) [7] and the comprehensive work on topological algebraic structures by Arhangel'skii and Tkachenko (2008) [8], provide the necessary rigor for such mixtures.

Despite these advancements, a unified framework integrating all three domains-algebra, rough set theory, and topology-for sophisticated structure like rings and fields remains largely unexplored. This paper bridges this gap by introducing a novel Topological-Rough Algebraic Framework. We cover the opinion of a topological approximation space to rings, constructing the triple $(R, \equiv_I, \tau_I)$ where an ideal I persuaded both an approximation relation and a topology. Within this space, we study rough set approximations and analyze their interaction with the algebraic structure, establishing new connections between these fundamental mathematical disciplines.

2. Openings

A unit provides a concise general idea of the important theories from R$o$ugh Set Theory, algebra, and topology on from the foundation of this work.

2.1. Rough Set Theory Fundamentals

2.1.1. Definition 2.1.1 Pawlak Approximation Space [1,2]

If U stay a non-empty space of objects for R is equality relative on U. The couple (U, R) remains named an rough calculation cosmos. The equality class of an element $x \in U$ is denoted by $[x]_R$,

For any subset $X \in U$, we define its lower approximation and upper approximation respectively as:

$\underline{R}(X) = \{x \in U: [x]_R \subseteq X\}$, $\overline{R}(X) = \{ x \in U: [x]_R \cap X \neq \emptyset$

The boundary area for X is given by $B.N_R(X) = \frac{\overline{R}(X)}{\underline{R}(X)}$. A set X is called definable or exact if $\underline{R}(X) = \overline{R}(X)$, otherwise, it is called a rough set [2].

2.2. Algebraic Foundations

2.2.1. Definition 2.2.2 (Ring and Ideal [4,9])

If disc $(R, +, \cdot)$ is a not-empty squad R prepared through dual processes (+ and ×):

$(R., +.)$ is an Abelian group

Multiplication distributes over toting

If Ideal I of a Ring R is a subset $I \subseteq R$ then:

$(I., +)$ is a subdivision of $(.R, +)$

$\forall r \in R$ & $a \in I, \exists\, r \cdot a \in I$ also $a \cdot r \in I$

2.2.2. Definition 2.2.3 Congruence Relation [4]

Let I be an ideal of a ring R. the congruence relation modulo I is defined for elements $a, b \in R$ as:

$a \equiv b \quad (mod\ I)$ if and only if $a - b \in I$. This relation is an equivalence classes are the cosets $a + I = \{a + i: i \in I\}$.

2.3. Topological Foundations

2.3.1. Definition 2.3.1 Topological Space [7]

A topological space is a duo (U, τ)everyplace U is a set for τ is a group of subgroup to U satisfying:

$$\emptyset, U \in \tau$$

The amalgamation of any collection of sets in τ be appropriate to τ.

The intersection of any finite collection of sets in τ fits to τ

2.4. The elements of τ are named open sets.

2.4.1. Definition 2.3.2 Base for Topology [7]

A family B of subsets of U is a base for a topology τ if:

For every $x \in U$,there exist $H \in B$ such that $x \in B$

If $x \in H_1 \cap H_2$ where $H_1, H_2 \in B$ then there exists $H_3 \in B$ such that

$$x \in H_3 \subseteq H_1 \cap H_2$$

2.4.2. Definition 2.3.3 Interior and Closure [7]

For a topological space (U, τ) and $X \subseteq U$:

The **Interior** of X, denoted $\boldsymbol{X}°$, be the union of wholly open sets restricted to X.

The **Closure** of X, denoted $\overline{\boldsymbol{X}}$, is the intersection of totally closed sets comprising X .

2.5. Previous Integration Results

2.5.1. Theorem 2.4.1 Wiweger [6]

Let (U, R) be a Pawlak approximation space. The family of definable sets forms a topology on U, and in this topology, for any $X \subseteq U$:

$$\underline{R}(X) = X°, \qquad \overline{R}(X) = \overline{\boldsymbol{X}}$$

2.5.2. Definition 2.4. 2 Rough Algebraic Structures[3,4]

A rough group is defined through approximations of group properties in an approximation space. Similarly, rough rings and rough Ideals extend these concepts to ring-theoretic structures using equivalence relations derived from the algebraic operations.

3. The Topological-Rough Algebraic Framework: Core Construction

This section presents the main contribution of this work: the construction of a unified topological-rough algebraic framework for rings and fields. We show how an ideal in a ring naturally induces both an approximation space and a topology.

3.1. Induced Approximation Space via Ring Ideals

3.1.1. Definition 3.1.1 Ideal-Induced Approximation Space

If R is a Ring and I an Ideal of R. The congruence relation module I defined by:

$$a \equiv_I b \quad \text{iff} \quad a - b \in I$$

The correspondence map on R. also duo $(R, \equiv_I)$ forms an estimate space induced by the ideal I. The equivalence class of an element $a \in R$ is the coset:

$$[a.]_{\equiv I} = a. + I = \{a. + i: i. \in I$$

The partition of R induced by this relation is the quotient ring $\frac{R}{I}$, whose elements are the cosets of I in R.

Example 3.1.2. Cons$i$der the r$i$ng of $i$ntegers .$\mathbb{Z}$ then the $i$deal

$I = 3\mathbb{Z} = \{\dots, -6, -3, 0, 3, 6, \dots\}$. The equivalence classes are:

$$[0] = \{\dots, -6, -3, 0, 3, 6, \dots\}$$

$$[1] = \{\dots, -5, -2, 1, 4, 7, \dots\}$$

$$[2] = \{\dots, -4, -1, 2, 5, 8, \dots\}$$

This forms the approximation space $(\mathbb{Z}, \equiv_{3\mathbb{Z}})$.

3.2. Topological Structure from Cosets

3.2.1. Theorem 3.2.1 (Coset Topology)

If R. be a r$i$ng and $I$is $i$deal in .R. The family of all cosets of .I:

$B_I = \{a + I : a \in R\}$ forms a base for a topology τ_I on R.

Proof: We verify the two conditions for a base:

Covering: $\forall x \in R$, then $x \in x + I \ \in H_I$ covers R.

Intersection condition: let $a +. I, b + I \ \in H_I$

and suppose $x \in (a + I) \cap (b + I)$. Then:

$x = a + i_1 = b + i_2$ for some $i_1, i_2 \in I$

This implies, $a - b = i_2 - i_1 \in I$. Therefore, $a + I, b + I$

So we can take $x + I. = .a + I = .b + I \subseteq (a + I) \cap (b + I)$

Hence, B_I is a base for a topology τ_I , which we call the coset topology induced by I.

3.2.2. Definition 3.2.2 (Topological-Rough Algebraic Space).

The triple $(R, \equiv_I, \tau_I)$ is called a topological-rough algebraic space induced by the ideal I.

3.3. Approximation Operators in the Induced Space

*3.3.1. Definition 3.3.1 (L**o**wer and Upp**e**r Appr**o**ximation)*

Aimed at every subset $X. \subseteq R$ in the topological-rough algebraic space $(R, \equiv_I, \tau_I)$, we define:

- **Lower Approximation:** $\underline{I}(X) = \{a \in R : a + I \subseteq X\}$,
- **Upper Approximation:** $\overline{I}(X) = \{\, a \in R : (a + I) \cap X \neq \emptyset$

3.3.2. Proposition 3.3.2.

In the topological-rough algebraic space $(R, \equiv_I, \tau_I)$, for any $X \subseteq R$: $\underline{I}(X) = X^\circ$, $\quad \overline{I}(X) = \overline{X}$. Where .$X^\circ$ and $\overline{X}$ denoted of X in the τ_I respectively. Evidence: This trails directly from theorem 2.4.1. and the construction of τ_I since the equivalence classes of $\equiv_I$.

Example 3.3.3: Continuing with example 3.1.2, let $X = \{0, 1, 4, 7\} \subseteq \mathbb{Z}$. Then:$\underline{I}(X) = \emptyset$ (no full coset is contained in X)

$\overline{I}(X) = [0] \cup [1] = \mathbb{Z} \setminus [2]$ (Since $[0] \cap X = [0]$,

$[1] \cap X = \{1, 4, 7\}, [2] \cap X = \emptyset$)

3.4. Extension to Fields

Remark 3.4.1. When R be a field and I is an $i$deal, the only possibilities are $I = \{0\}$ or $I = R$. Therefore:

If $I = \{0\}$, then $\equiv_I$ is the identity relation and τ_I is the discrete topology

If $I = R$ then there is only one equivalence class and τ_I is the indiscrete topology .

From more interesting structures in fields, one may consider subfield-based equivalence relations or work with valuation rings in algebra number theory.

3.5. First Properties

Proposition 3.5.1: The Topology τ_I is:

Totally disconnected if $I \neq R$.

Homogeneous (any two points have homeomorphic neighborhoods)

Translation-invariant: For any $x \in R$, the translation map $x \mapsto x + r$ is homeomorphism.

Proof:

Each coset is clopen (both open and closed) in τ_I, making the space totally disconnected.

The algebraic structure ensures homogeneity.

Ring addition preserves cosets, making translations homeomorphisms.

3.6. Properties and Characterization Theorems

This section investigates the fundamental properties of our topological-rough algebraic framework and establishes characterization theorems for rough sets within this structure.

3.6.1. Theorem 4.1.1 (Exactness Characterization)

Let $(R, \equiv_I, \tau_I)$ be a topological-rough algebraic space and $X \subseteq R$, the following are equivalent:

X is exact (i.e., $\underline{I}(X). =. \overline{I}(X)$.

X is a union of cosets of I.

X is clopen in τ_I.

Proof: $(\boldsymbol{a}) \Leftrightarrow (\boldsymbol{b})$

From Definition 3.3.1. $\underline{I}(X) = \overline{I}(X)$ iff for every $a \in R$, either $a + I \subseteq X$ or $(a + I) \cap X = \emptyset$. This occurs precisely when X is a union of complete cosets.(b) $\Leftrightarrow$ $(\boldsymbol{c})$: Inτ_I, every coset is clopen. Any union of clopen sets is clopen. Conversely, any clopen set in τ_I must be a union of the connected components (cosets).

Example 4.1.2: Consider the ring $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ and ideal $I = \{0, 3\}$ (which is the principal ideal generated by 3). The cosets are:

-$[0] = \{0, 3\}, [1] = \{1, 4\}, [2] = \{2, 5\}$

Let $X = \{0, 1, 3, 4\}$. Then : $X = [0] \cup [1]$ (union of cosets),

$\underline{I}(X) = \overline{I}(X) = X$ (exact set), X *is clopen in* τ_I.

3.7. Roughness Measures and Boundary Analysis

Definition 4.2.1 (Algebraic Accuracy Measure). For any subset $X \subseteq R$ in $(R, \equiv_I, \tau_I)$, we define the algebraic accuracy measure: $\alpha_I(X) = \frac{|\underline{I}(X)|}{|\overline{I}(X)|}$With the convention that $\alpha_I(X) = 1 \; when \; \overline{I}(X) = \emptyset$

Proposition 4.2.2: The algebraic accuracy measure satisfies:

$0 \leq \alpha_I(X) \leq 1 \; \forall \; X \subseteq R.$

$\alpha_I(X) = 1$ if and only if X is exact.

$$\alpha_I(X) = 0 \iff \overline{I}(X) = \emptyset.$$

Example 4.2.3: Continuing with $\mathbb{Z}_6$ and ideal $I = \{0, 3\}$, let $Y = \{0, 1, 2\}$.

$\underline{I}(Y) = \emptyset$ (no full coset contained in Y)

$$\mathbb{Z}_6 = [0] \cup [1] \cup [2] = I(Y)$$

$$\alpha_I(Y) = \frac{0}{6} = 0$$

$BN_I(Y) = \mathbb{Z}_6$ (entire space is boundary)

Theorem 4.2.4 (Boundary Structure Theorem). The boundary region of any $X \subseteq R$ satisfies: $I: (a. + I.) \cap . X \neq \emptyset$ and $(a. + . I \;) \cap X^c \neq \emptyset\}$, that is, boundary consists precisely of those cosets that property intersect both X and its complement.

Proof: Follows directly from the definitions and the fact that cosets are the connected components of the space.

3.8. Ideal Operation and Topological Consequences

3.8.1. Theorem 4.3.1 (Nested Ideals and Topological Refinement).

If I, J are $i$deals of R with $J . \subseteq . I$. Then:

$\equiv_I$ is a coarsening $of \equiv_J$ (i, e,, $a \equiv_J b \Longrightarrow a \equiv_I b$)

τ_I is coarser than τ_J (i. e., $\tau_I \subseteq \tau_J$)

for any $X \subseteq R, \underline{I}(X) \; \subseteq \underline{J}(X) \; \subseteq X \subseteq \; \overline{J}(X) \subseteq \underline{I}(X)$)

proof;

(1). If $a - b \in J \subseteq I$, then $a - b \in I$

(2). Each coset of I is a union of cosets of J

(3). Follows from the set inclusions of approximations

Example 4.3.2: In $\mathbb{Z}_{12}$, consider ideals $I = \{0, 3, 6, 9\}$ and $J = \{0,6\}$ with $J \subseteq \; I$:

Cosets of I: $[0] = \{0, 3, 6, 9\}, [1] = \{1, 4, 7, 10\}, [2] = \{2, 5, 8, 11\}$

Cosets of J: $[0] = \{0, 6\}, [1] = \{1, 7\}, [2] = \{2, 8\}, [3] = \{3,9\}$,

$$[4] = \{4,10\}, [5] = \{5, 11\}$$

Let, $X = \{0,1,2,6,7,8\}$. Then:

$$\mathbb{Z}_{12} = \bar{I}(X), \quad \underline{I}(X) = \emptyset$$

$$\{0,1,2,6,7,8\} = [0] \cup [1] \cup [2] = \underline{J.}(X) \to \underline{J.}(X) = .\bar{J}(X)$$

Thus , $\underline{I}(X) \subset \underline{J.}(X) = X = \bar{J}(X) \subset \bar{I}(X)$)

3.9. Special Cases and Classification

3.9.1. Theorem 4.4.1 (Maximal Ideal Characterization).

If I be a greatest $i$deal of R, at that time:

The proportion r$i$ng R/I be a field.

The topology τ_I has the property that every non-empty open set is dense or nowhere dense

For any $X \subseteq R$, either X is exact or has maximal boundary

3.9.2. Theorem 4.4.2 (Prime Ideal and Separation Properties)

The I is prime $i$deal to an integral domain .R, now the topological space (R, τ_I) is:

$$\tau_0 \ but\ not\ \tau_1\ if\ I \neq \{0\}$$

Totally disconnected

Locally compact if R is equipped with an appropriate algebraic topology

Example 4.4.3: In $\mathbb{Z}$, the ideal $I = P^{\mathbb{Z}}$ for prime P

I is maximal and prime

The coset topology τ_I has exactly P clopen sets (the cosets)

For any subset $X \subseteq \mathbb{Z}$, the boundary $BN_I(X)$ is either empty or the whole space for each coset

These results demonstrate the rich interplay between algebraic properties of ideals and topological characteristics of the induced approximation spaces, proving a powerful classification system for rough sets in algebraic contexts.

4. Application and Computational Examples

4.1. Application in Coding Theory

4.1.1. Theorem 5.1.1 (Error-Detecting Codes Characterization)

Let $C \subseteq \mathbb{F}_q^n$ be a linear code filed finite filed$\mathbb{F}_q$with parity-check matrix H. Consider the ideal $I = \{x \in \mathbb{F}_q^n : H_x^\tau = 0\}$ (the code itself). Then in the space $(\mathbb{F}_q^n, \equiv_I, \tau_I)$:

A received vector y contains detectable error iff $y \in BN_I(C)$

The error pattern is correctable iff y belong to a unique coset of C

Proof:

The cosets of C correspond to distains syndromes in coding theory . The boundary region represents detectable but uncorrectable errors.

Example 5.1. 2: consider the binary linear code $C = \{000, 111 \subseteq \mathbb{F}_2^3\}$ with ideal $I = C$. Consider the binary repetition code $C = \{000, 111\} \subseteq \mathbb{F}_2^3$. Which is a linear code with minimum distance 3. This code can correct single-bit errors. Let us take the ideal $I = C$ itself. The coset decomposition of $\mathbb{F}_2^3$ by is given by:

$$C_0 = [000] = C = \{000, 111\}$$

$$C_1 = [001] = \{001, 011\}$$

$$C_2 = [010] = \{010, 101\}$$

$$C_3 = [100] = \{100, 110\}$$

each coset corresponds to a unique syndrome in standard coding theory. Suppose the transmitted code word is $C = 000$ and the received vector is $y = 101$ due to a 2-bit error.

The received vector y belongs to coset $C_2 = [010] = \{010, 101\}$

The lower approximation is $\underline{I}(\{101\}) = \emptyset$ because no entire coset is contained in the singleton set $\{101\}$.

The upper approximation is $\overline{I}(\{101\}) = C_2 = \{010, 101\}$ because this coset has a non-empty intersection with $\{101\}$.

4.2. Cryptographic Analysis Application

4.2.1. Theorem 5.2.1 (side-Channel Attack Resistance)

Let R be a ring used in cryptographic operations and I an ideal representing observable power consumption patterns/. A cryptographic implementation is resistant to side-channel attacks if for all secret keys k: $\alpha_I(\{k\}) = 0$. That is, the lower approximation of any singleton key sets empty.

Proof: If $\alpha_I(\{k\}) = 0$, then the key cannot be precisely identified from the observable patterns (cosets), proving resistance to statistical analysis (RSA).

Example 5.2.2.

Consider RSA operations in $\mathbb{Z}_{pq}$with p, q primes. Let ideal $I = p\mathbb{Z}_{pq}$. The cosets are:

$$[0] = \{0.p, p^2, \dots, (1-q).p\}$$

$$[1.] = \{1, p+1, 1+.p^2, \dots, (1-q)(1+.p)\}$$

…

…

$$[.1-p] = \{1-.p, 1-p^2, \dots, (1-pq)\}$$

For any secret key k, $\underline{I}(\{k\}) = \emptyset$ since no coset is a singleton. The boundary $BN_I(k)$ contains the enter coset$[k]$, making precise identification impossible from this coarse observation.

4.3. Data Mining with Algebraic Structure

4.3.1. Algorithm 5.3.1 (Rough Clustering Algebraic Data)

- **Input:** Dataset $D \subseteq R$ with algebraic structure, ideal I
- **Output:** Clusters with roughness measures

Construct the topological-rough space $(R, \equiv_I, \tau_I\ )$

For each data point $x \in D$, identity its coset $[x] = x + 1$

Merge cosets that share data points(upper approximation)

Identity pure cosets containing only one cluster (lower approximation)

Compute $\alpha_I(C)$ for each cluster C

Example 5.3.2. Customer data encoded as elements of $\mathbb{Z}_{100}$ with ideal $I = 10\mathbb{Z}_{100}$ (grouping by spending range). Cosets: $[0], [1], [2], \dots, [9]$ representing spending ranges $0-9, 10-19, \dots, 90-99$.

Data: D={15, 25, 35, 16, 26, 45}

Cosets: $[5] = \{15, 25, 35, 45\}, [6] = \{16, 26\}$

Cluster:$C_1 = \{15, 25, 35, 45\}$: $\underline{I}(C_1) = C_1, \overline{I}(C_1) = C_1 \Longrightarrow \alpha_I(C_1) = 1$ (exact)

Cluster: $C_2 = \{16, 26\}$: $\underline{I}(C_2) = C_2, \overline{I}(C_2) = C_2 \subseteq \alpha_I(C_2) = 1$ (exact)

4.4. Network Security and Intrusion Detection

4.4.1. Theorem 5.4.1 (Anomaly Detection Characterization)

In a network traffic model with ring structure R and ideal I representing normal behavior patterns, an intrusion is detected when: traffic pattern $t \in BN_I(N)$

Where, $N \subseteq R$ is the set of normal traffic patterns.

Example 5.4.2: Consider network packets encoded in $\mathbb{Z}_{256}$ with ideal $I = 4\mathbb{Z}_{256}$ (grouping by packet size modulo 4). Normal traffic $N = [0] \cup [1] = \{0, 1, 4, 5, 8, 9, \dots\}$.

Normal packet: $t = 9 \in [1] \subseteq N \Longrightarrow \underline{I}(\{9\}) = \{9\}$, exact

Anomalous packet: $t = 3 \in [3], [3] \cap N = \emptyset \Longrightarrow \underline{I}(\{3\}) = \emptyset$

$\overline{I}(\{3\}) = [3] \Longrightarrow t \in BN_I(N)$, intrusion detected

4.5. Computational Complexity Analysis

Proposition 5.5.1. For a finite ring R of order n and ideal I of order m:

Construction of τ_I: $O(n)$ time

Lower/upper approximation of $X \subseteq R$: $O(n)$ time

Accuracy measure computation: $O(n)$time

Proof:

The coset enumeration and set operations can be implemented efficiently using the algebra ic structure, proving polynomial-time complexity for all major operation.

These applications demonstrate that our topological-rough algebraic framework provides not only theoretical incising but also practical algorithms for real-world problems across multiple algebraic structure, rough set approximations, and topological properties.

5. Conclusion

A New Hybrid Construction: The article manages to develop a Topological-Rough Algebraic Framework by combining Rough Set Theory, Algebra (rings/ideals), and General Topology.

Building General Case Construction: It is constructed around the triple **(R, ≡_I, τ_I)** Finding Construction A) A patition by an ideal I induces the analogue of an equivalence relation (coset partition) of the ring R B) An ideal I as the statistic of the thrown-communicator gathers a topology τI on R.

Theoretical Deliberations: It gives a comprehensive account of this framework, to define rough/exact sets and made connections among topological characteristics and algebraic format.

Practical Applications: The framework has been applied in coding theory, cryptography and data mining.

Significant Generalization: This is the extension of the rough set theory of Pawlak to complex algebraic objects, which provides a new and powerful mathematical analysis instrument.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

References

[1] Pawlak, Z. (1982). Rough Sets. International Journal of Computer & Information Sciences, 11(5), 341-356.

[2] Pawlak, Z., & Skowron, A. (2007). Rudiments of rough sets. Information Sciences, 177(1), 3-27.

[3] Biswas, R., & Nanda, S. (1994). Rough Groups and Rough Subgroups. Bulletin of the Polish Academy of Sciences Mathematics 42(3), 251-254.

[4] Davvaz, B., (2019). Roughness in rings. Information Sciences, 164(1-4), 147-163.

[5] Miao, D., et al. (2021). Rough sets based on ideals in lattices International Journal of Approximate Reasoning, 139, 54-68.

[6] Wiweger, A.,(1989). On topological rough sets. Bulletin of Polish Academy of Sciences: Mathematics 37(1-6), 89-93.

[7] Munkres, J. R. (2000). Topology(2nded). Prentice Hall.

[8] Arhangel'skii, A., & Tkachenko, M. (2008). Topological Groups and Related Structures, Atlantis Press.

[9] Hungerford, T. W.(1974). Algebra. Springer-Veriag.